Colour conversions for DCP creation

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1 Overview

The process of conversion from RGB to XYZ is:

1. Convert to linear RGB (i.e. apply a gamma curve, or at least an approximation to one).
2. Convert to XYZ (preserving the white point).
3. Adjust the white point.
4. Normalize values.
5. Convert to non-linear XYZ (i.e. apply a gamma curve)

2 Convert to linear RGB

This is done using either a 'pure' gamma function, so that for some colour value C_i the output colour C_o is given by

$$C_o = C_i^\gamma \tag{1}$$

or a modified function of the form

$$C_o = \begin{cases} \frac{C_i}{K} & C_i \leq C_t \\ \left(\frac{C_i + A}{1 + A}\right)^\zeta & C_i > C_t \end{cases} \tag{2}$$

where K, A, C_t and ζ are constants. This modified function approximates a 'pure' gamma function but changes the output for small inputs.

3 Convert to XYZ

This is done by multiplying the colours by a 3×3 matrix. This matrix depends on the chromaticities of the RGB primaries and the white point. Note that these are the same for BT709 and sRGB, so the corresponding matrices are the same.

The chromaticities for sRGB and BT709 are shown in Table **??**. The white point is ($x = 0.3127$, $y = 0.329$); this is called D65.

	x	y
Red	0.64	0.33
Green	0.3	0.6
Blue	0.15	0.06

Table 1: RGB chromaticities for sRGB and BT709

Let

$$D = \begin{vmatrix} R_x - W_x & W_x - B_x \\ R_y - W_y & W_y - B_y \end{vmatrix} \tag{3}$$

$$E = \begin{vmatrix} W_x - G_x & R_x - W_x \\ W_y - G_y & R_y - W_y \end{vmatrix} \tag{4}$$

$$F = \begin{vmatrix} W_x - G_x & W_x - B_x \\ W_y - G_y & W_y - B_y \end{vmatrix} \tag{5}$$

$$P = R_y + G_y \frac{D}{F} + B_y \frac{E}{F} \tag{6}$$

where:

- R_x, R_y: red point; $R_z = 1 - R_x - R_y$
- G_x, G_y: green point; $G_z = 1 - G_x - G_y$
- B_x, B_y: blue point; $B_z = 1 - B_x - B_y$
- W_x, W_y: white point

Then the conversion matrix **C** is as follows:

$$\mathbf{C} = \frac{1}{P}\begin{bmatrix} R_x & G_x\frac{D}{F} & B_x\frac{E}{F} \\ R_y & G_y\frac{D}{F} & B_y\frac{E}{F} \\ R_z & G_z\frac{D}{F} & B_z\frac{E}{F} \end{bmatrix} = \begin{bmatrix} 0.4123908 & 0.3575843 & 0.1804808 \\ 0.2126390 & 0.7151687 & 0.0721923 \\ 0.0193308 & 0.1191948 & 0.9505322 \end{bmatrix} \tag{7}$$

Then to convert RGB to XYZ we do

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{C}\begin{bmatrix} R \\ G \\ B \end{bmatrix} \tag{8}$$

i.e.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4123908 & 0.3575843 & 0.1804808 \\ 0.2126390 & 0.7151687 & 0.0721923 \\ 0.0193308 & 0.1191948 & 0.9505322 \end{bmatrix}\begin{bmatrix} R \\ G \\ B \end{bmatrix} \tag{9}$$

Note: there is also a CIE definition of the D65 white point as ($x = 0.31272$, $y = 0.32903$), which we do not use.

4 Adjust the white point

If required we can adjust the white point of the colours from one white point $\mathbf{S_1}$ to another $\mathbf{S_2}$. This is done by multiplication by a Bradford matrix, **B**. To calculate it, we start with the Bradford chromatic adaption transform matrix **M**, taken from Süsstrunk et al., *Chromatic adaption performance of different RGB sensors*, IS&T/SPIE Electronic Imaging, SPIE Vol. 4300 (2001).

$$\mathbf{M} = \begin{bmatrix} 0.8951 & 0.2664 & -0.1614 \\ -0.7502 & 1.7135 & 0.0367 \\ 0.0389 & -0.0685 & 1.0296 \end{bmatrix} \tag{10}$$

The inverse of $\mathbf{M}$ is

$$\mathbf{M}^{-1} = \begin{bmatrix} 0.9869929055 & -0.1470542564 & 0.1599626517 \\ 0.4323052697 & 0.5183602715 & 0.0492912282 \\ -0.0085286646 & 0.0400428217 & 0.9684866958 \end{bmatrix} \tag{11}$$

Next, compute $\mathbf{G}$ and $\mathbf{H}$ as follows

$$\mathbf{G} = \mathbf{M} \begin{bmatrix} \dfrac{S_{1x}}{S_{1y}} \\ 1 \\ \dfrac{1 - S_{1x} - S_{1y}}{S_{1y}} \end{bmatrix} \tag{12}$$

$$\mathbf{H} = \mathbf{M} \begin{bmatrix} \dfrac{S_{2x}}{S_{2y}} \\ 1 \\ \dfrac{1 - S_{2x} - S_{2y}}{S_{2y}} \end{bmatrix} \tag{13}$$

Then the Bradford matrix $\mathbf{B}$ is given by

$$\mathbf{B} = \mathbf{M}^{-1} \left(\begin{bmatrix} \dfrac{H_1}{G_1} & 0 & 0 \\ 0 & \dfrac{H_2}{G_2} & 0 \\ 0 & 0 & \dfrac{H_3}{G_3} \end{bmatrix} \mathbf{M} \right) \tag{14}$$

$$\tag{15}$$

5 Normalize values

Here we just multiply all colour values by the constant N where

$$N = \frac{48}{52.37} \tag{16}$$

6 Convert to non-linear XYZ

This is a gamma correction of 1/2.6, so that for some colour value C_i the output colour C_o is given by

$$C_o = C_i^{1/2.6} \tag{17}$$

7 Converting from a colour matrix to chromaticities

To get back from a colour matrix **C** to the chromaticities:

$$R_x = \frac{C_{11}}{C_{11} + C_{21} + C_{31}} \tag{18}$$

$$R_y = \frac{C_{21}}{C_{11} + C_{21} + C_{31}} \tag{19}$$

$$G_x = \frac{C_{12}}{C_{12} + C_{22} + C_{32}} \tag{20}$$

$$G_y = \frac{C_{22}}{C_{12} + C_{22} + C_{32}} \tag{21}$$

$$B_x = \frac{C_{13}}{C_{13} + C_{23} + C_{33}} \tag{22}$$

$$B_y = \frac{C_{23}}{C_{13} + C_{23} + C_{33}} \tag{23}$$

$$W_x = \frac{C_{11} + C_{12} + C_{13}}{C_{11} + C_{12} + C_{13} + C_{21} + C_{22} + C_{23} + C_{31} + C_{32} + C_{33}} \tag{24}$$

$$W_y = \frac{C_{21} + C_{22} + C_{23}}{C_{11} + C_{12} + C_{13} + C_{21} + C_{22} + C_{23} + C_{31} + C_{32} + C_{33}} \tag{25}$$